

Higgs boson decay to $\mu\bar{\mu}\gamma$

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The Higgs boson decay, $H \rightarrow \mu\bar{\mu}\gamma$, is studied in the Standard Model at the tree and one-loop levels. It is shown that for Higgs boson masses above 110 GeV, the contribution to the radiative width from the one-loop level exceeds the contribution from the tree level, and for Higgs boson masses above 140 GeV, it even exceeds the contribution from the tree level decay $H \rightarrow \mu\bar{\mu}$. We also show that the contributions to the radiative decay width from the interference terms between the tree and one-loop diagrams are negligible.

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I. INTRODUCTION

For intermediate mass Higgs (H) bosons, the decay $H \rightarrow \gamma\gamma$ is usually viewed as the discovery mode [1], although decay modes such as $H \rightarrow Z\gamma$ and $H \rightarrow b\bar{b}$ are often considered. Here, motivated by recent studies of muon colliders [2], we examine Higgs decays into muons accompanied by a photon. At muon colliders, these decays will, at the very least, represent a radiative correction to the measurement of the width $\Gamma(H \rightarrow \mu\bar{\mu})$. For certain values of the Higgs boson mass, m_H , this radiative process can be quite large due to one-loop corrections. As shown in a previous study of the decays $H \rightarrow f\bar{f}\gamma$ [3], where f is a light fermion, the dominant contributions come from the one-loop level when m_f is negligible. The one-loop decay channel calculation is related to that of the scattering process $e\bar{e} \rightarrow H\gamma$ [4,5]. For muons, the dominance of the one-loop calculation must be reexamined.

Due to the relatively large mass of muon ($m_\mu \gg m_e$), the Higgs-boson-muon coupling is sufficiently large to make the tree level contribution to the decay $H \rightarrow \mu\bar{\mu}\gamma$ significant. Our calculations show that the tree level amplitudes are dominated by the muon helicity non-flip terms, while the dominant contributions to the one-loop amplitudes come from the muon helicity flip terms. Therefore, the contributions to the radiative decay width from the interference terms between the tree and one-loop diagrams turn out to be negligible.

In the next section, we present general discussion of the kinematics of the Higgs boson decay width and muon invariant-mass distribution for $H \rightarrow \mu\bar{\mu}\gamma$, and the cuts imposed on the μ , $\bar{\mu}$, and γ . Section III gives the tree level amplitudes for the decay $H \rightarrow \mu\bar{\mu}\gamma$ and a summary of the results for the one-loop calculation. Section IV contains the combined contributions from the tree and one-loop levels to the decay of the Higgs boson. This is followed by a summary which includes a discussion of possibility of using the decay $H \rightarrow \mu\bar{\mu}\gamma$ as a probe of the Higgs boson coupling to the top quark.

II. HIGGS BOSON DECAY WIDTHS

The muon-invariant mass distribution $d\Gamma/dm_{\mu\bar{\mu}}$ and the width Γ for the decay $H \rightarrow \mu\bar{\mu}\gamma$ are given by

$$\frac{d\Gamma}{dm_{\mu\bar{\mu}}} = \frac{1}{128\pi^3} \frac{m_{\mu\bar{\mu}}}{m_H^3} \int_{(m_{\mu\bar{\mu}}^2)_{\min}}^{(m_{\mu\bar{\mu}}^2)_{\max}} dm_{\mu\bar{\mu}}^2 \sum_{\text{spin}} |\mathcal{M}|^2, \quad (1)$$

$$\Gamma = \int_{(m_{\mu\bar{\mu}}^2)_{\min}}^{(m_{\mu\bar{\mu}}^2)_{\max}} dm_{\mu\bar{\mu}}^2 \frac{1}{2m_{\mu\bar{\mu}}} \frac{d\Gamma}{dm_{\mu\bar{\mu}}}, \quad (2)$$

with the Lorentz-invariant amplitude \mathcal{M} specified in the next section. The invariant masses $m_{\mu\bar{\mu}}$ and $m_{\mu\gamma}$ are related to the Mandelstam variables s , t , and u , by $s = m_{\mu\bar{\mu}}^2 = (p_\mu + p_{\bar{\mu}})^2$, $t = m_{\mu\gamma}^2 = (p_\mu + p_\gamma)^2$, and $u = m_{\bar{\mu}\gamma}^2 = (p_{\bar{\mu}} + p_\gamma)^2$. Here, p_μ , $p_{\bar{\mu}}$, and p_γ are the 4-momenta for the μ , $\bar{\mu}$, and γ , respectively. The limits on the $dm_{\mu\bar{\mu}}^2$ and $dm_{\mu\gamma}^2$ integrations, without imposing any cuts, are

$$(m_{\mu\gamma}^2)_{\min} = m_\mu^2 + \frac{1}{2}(m_H^2 - m_{\mu\bar{\mu}}^2)(1 - \beta), \quad (3)$$

$$(m_{\mu\gamma}^2)_{\max} = m_\mu^2 + \frac{1}{2}(m_H^2 - m_{\mu\bar{\mu}}^2)(1 + \beta), \quad (4)$$

$$(m_{\mu\bar{\mu}}^2)_{\min} = 4m_\mu^2, \quad (5)$$

$$(m_{\mu\bar{\mu}}^2)_{\max} = m_H^2, \quad (6)$$

with $\beta = \sqrt{1 - 4m_\mu^2/m_{\mu\bar{\mu}}^2}$. Because the full phase space is not experimentally accessible, we impose the following cuts: $m_{\mu\bar{\mu}}^2 \geq (m_{\mu\bar{\mu}}^2)_{\text{cut}}$, $m_{\mu\gamma}^2 \geq (m_{\mu\gamma}^2)_{\text{cut}}$, $m_{\bar{\mu}\gamma}^2 \geq (m_{\bar{\mu}\gamma}^2)_{\text{cut}}$, $E_\mu \geq (E_\mu)_{\text{cut}}$, $E_{\bar{\mu}} \geq (E_{\bar{\mu}})_{\text{cut}}$, and $E_\gamma \geq (E_\gamma)_{\text{cut}}$. Here, E_μ , $E_{\bar{\mu}}$, and E_γ are the muon, anti-muon, and photon energies, respectively, in the center of mass of the Higgs boson. For our present work, we restrict these cuts to: $(m_{\mu\bar{\mu}}^2)_{\text{cut}}, (m_{\mu\gamma}^2)_{\text{cut}}, (m_{\bar{\mu}\gamma}^2)_{\text{cut}} \gg 4m_\mu^2$, and $(E_\mu)_{\text{cut}}, (E_{\bar{\mu}})_{\text{cut}}, (E_\gamma)_{\text{cut}} \gg 2m_\mu$.

With these cuts imposed, the limits on the $dm_{\mu\gamma}^2$ and $dm_{\mu\bar{\mu}}^2$ integrations in Eqs. (1) and (2) are modified to

$$(m_{\mu\gamma}^2)_{\min} = \max[(m_{\mu\gamma}^2)_{\text{cut}}, t_1], \quad (7)$$

$$(m_{\mu\gamma}^2)_{\max} = \min[t_2, t_3], \quad (8)$$

$$(m_{\mu\bar{\mu}}^2)_{\min} = \max[(m_{\mu\bar{\mu}}^2)_{\text{cut}}, s_1], \quad (9)$$

$$(m_{\mu\bar{\mu}}^2)_{\max} = \min[s_2, s_3], \quad (10)$$

where

$$t_1 = 2m_H(E_\mu)_{\text{cut}} - m_{\mu\bar{\mu}}^2, \quad (11)$$

$$t_2 = m_H^2 - 2m_H(E_{\bar{\mu}})_{\text{cut}}, \quad (12)$$

$$t_3 = m_H^2 - m_{\mu\bar{\mu}}^2 - (m_{\bar{\mu}\gamma}^2)_{\text{cut}}, \quad (13)$$

$$s_1 = 2m_H(E_\mu)_{\text{cut}} + 2m_H(E_{\bar{\mu}})_{\text{cut}} - m_H^2, \quad (14)$$

$$s_2 = m_H^2 - 2m_H(E_\gamma)_{\text{cut}}, \quad (15)$$

$$s_3 = m_H^2 - (m_{\mu\gamma}^2)_{\text{cut}} - (m_{\bar{\mu}\gamma}^2)_{\text{cut}}. \quad (16)$$

Notice that in order to implement and assign values to these cuts, one needs to observe the following constraints

$$(m_{\mu\bar{\mu}}^2)_{\text{cut}} + 2m_H(E_\gamma)_{\text{cut}} \leq m_H^2, \quad (17)$$

$$(m_{\mu\gamma}^2)_{\text{cut}} + 2m_H(E_{\bar{\mu}})_{\text{cut}} \leq m_H^2, \quad (18)$$

$$(m_{\bar{\mu}\gamma}^2)_{\text{cut}} + 2m_H(E_\mu)_{\text{cut}} \leq m_H^2, \quad (19)$$

$$(m_{\mu\bar{\mu}}^2)_{\text{cut}} + (m_{\mu\gamma}^2)_{\text{cut}} + (m_{\bar{\mu}\gamma}^2)_{\text{cut}} \leq m_H^2, \quad (20)$$

$$(E_\mu)_{\text{cut}} + (E_{\bar{\mu}})_{\text{cut}} + (E_\gamma)_{\text{cut}} \leq m_H, \quad (21)$$

$$(m_{\mu\bar{\mu}}^2)_{\text{cut}}, (m_{\mu\gamma}^2)_{\text{cut}}, (m_{\bar{\mu}\gamma}^2)_{\text{cut}} \gg 4m_\mu^2, \quad (22)$$

$$(E_\mu)_{\text{cut}}, (E_{\bar{\mu}})_{\text{cut}}, (E_\gamma)_{\text{cut}} \gg 2m_\mu. \quad (23)$$

In our present calculations for the decay width $\Gamma(H \rightarrow \mu\bar{\mu}\gamma)$ and the $\mu\bar{\mu}$ -invariant mass distribution $d\Gamma(H \rightarrow \mu\bar{\mu}\gamma)/dm_{\mu\bar{\mu}}$, we choose the following set of cuts:

$$(E_\mu)_{\text{cut}} = (E_{\bar{\mu}})_{\text{cut}} = (E_\gamma)_{\text{cut}} = 1 \text{ GeV}, \quad (24)$$

$$(m_{\mu\bar{\mu}}^2)_{\text{cut}} = (m_{\mu\gamma}^2)_{\text{cut}} = (m_{\bar{\mu}\gamma}^2)_{\text{cut}} = 25m_\mu^2. \quad (25)$$

These cuts facilitate the experimental tagging of μ , $\bar{\mu}$, and γ . They provide minimum opening angles between μ , $\bar{\mu}$, and γ , and also avoid the collinear and infrared divergences. The cuts also help discriminate the non-back-to-back $\mu\bar{\mu}$ pairs of the decay $H \rightarrow \mu\bar{\mu}\gamma$ from the back-to-back $\mu\bar{\mu}$ pairs of the decay $H \rightarrow \mu\bar{\mu}$. In principle, all the muons and photons of the decays $H \rightarrow \mu\bar{\mu}\gamma$, $H \rightarrow \mu\bar{\mu}$, and $H \rightarrow \gamma\gamma$ can be identified.

III. INVARIANT AMPLITUDES

A. Tree Level Amplitudes

The leading order muon helicity non-flip amplitudes for the decay $H \rightarrow \mu\bar{\mu}\gamma$ can be written as

$$\mathcal{M}_{\lambda\bar{\lambda}\lambda_\gamma}^{\text{tree}} = i \frac{egm_\mu}{\sqrt{2}tu m_W} \begin{cases} +s & , \lambda\bar{\lambda}\lambda_\gamma = --+, \\ -s & , \lambda\bar{\lambda}\lambda_\gamma = +- -, \\ +m_H^2 & , \lambda\bar{\lambda}\lambda_\gamma = +++ , \\ -m_H^2 & , \lambda\bar{\lambda}\lambda_\gamma = --- , \end{cases} \quad (26)$$

where $\lambda = \pm 1/2$, $\bar{\lambda} = \pm 1/2$, and $\lambda_\gamma = \pm 1$ are the helicities of the μ , $\bar{\mu}$, and γ , respectively and we show only the signs of the helicities in Eq. (26).

The leading order muon helicity flip amplitudes are

$$\mathcal{M}_{\lambda\bar{\lambda}\lambda_\gamma}^{\text{tree}} = i \frac{egm_\mu^2\sqrt{s}}{\sqrt{2}m_W} \frac{st + su + t^2 + u^2}{(s+t)(s+u)} \times \begin{cases} 1/t & , \lambda\bar{\lambda}\lambda_\gamma = -++ , \\ 1/t & , \lambda\bar{\lambda}\lambda_\gamma = +- -, \\ 1/u & , \lambda\bar{\lambda}\lambda_\gamma = -+- , \\ 1/u & , \lambda\bar{\lambda}\lambda_\gamma = +- -. \end{cases} \quad (27)$$

In the Eqs. (26) and (27), we have kept only the leading orders in m_μ , since we are assuming $s, t, u \gg 4m_\mu^2$ and $E_\mu, E_{\bar{\mu}}, E_\gamma \gg 2m_\mu$. This is consistent with the cuts of $s, t, u \geq 25m_\mu^2$ and $E_\mu, E_{\bar{\mu}}, E_\gamma \geq 1 \text{ GeV}$, that we impose on the calculations of the decay width and the invariant mass distribution.

It can be seen from the Eqs. (26) and (27) that the helicity flip amplitudes have an extra factor of m_μ . This is an expected behavior. As discussed in the Ref. [6], the leptonic-current factors in the amplitudes, are proportional to linear combinations of $\bar{u}(p_\mu)v(p_{\bar{\mu}})$ and $\bar{u}(p_\mu)\sigma_{\alpha\beta}v(p_{\bar{\mu}})$. The muon helicity non-flip contributions from these terms survive in the $m_\mu \rightarrow 0$ limit, while the corresponding muon helicity flip contributions are proportional to m_μ , and vanish in this limit. For the tree level amplitudes, we may, therefore, neglect the contribution from the muon helicity flip amplitudes, and consider only the contribution from the muon helicity non-flip amplitudes to the decay width and the invariant mass distribution.

B. One-Loop Results

Contributions of the one-loop amplitudes to the decay $H \rightarrow \mu\bar{\mu}\gamma$ arise from the diagrams illustrated in the Fig. 1. The explicit expressions for the amplitudes corresponding to these diagrams are given in the Ref. [3]. As discussed in the Ref. [6], the leptonic-current factors in these amplitudes, are proportional to linear combinations of $\bar{u}(p_\mu)\gamma_\alpha v(p_{\bar{\mu}})$ and $\bar{u}(p_\mu)\gamma_\alpha\gamma_5 v(p_{\bar{\mu}})$. The muon helicity flip contributions from these terms survive in the $m_\mu \rightarrow 0$ limit, while the muon helicity non-flip contributions, which are proportional to m_μ , do not. In this case, we may neglect the contribution from the muon helicity non-flip amplitudes, and consider only the contribution from the muon helicity flip amplitudes to the decay width and the invariant mass distribution.

The expression for $\sum_{\text{spin}} |\mathcal{M}|^2$, given by Eq. (8) of the Ref. [3], together with Eq. (2), can be used to calculate the one-loop contribution to $\Gamma(H \rightarrow \mu\bar{\mu}\gamma)$. The results are illustrated in the Fig. 2. In this figure, contributions to the width from the triangle and box diagrams of Fig. 1 are shown separately. The combined contributions from the Z and photon poles in the Fig. 1(a) con-

stitute almost the entire contribution of all diagrams in the Fig. 1. Notice that for Higgs boson masses not too much above 100 GeV, the photon pole makes substantial contribution. Therefore, the simple estimate of the decay width $\Gamma(H \rightarrow \mu\bar{\mu}\gamma)$, obtained by multiplying the width $\Gamma(H \rightarrow Z\gamma)$ by the branching ratio $B(Z \rightarrow \mu\bar{\mu})$, will receive large correction due to the photon pole diagram. However, for the $m_H \gtrsim 130$ GeV, it is the Z pole that gives most of the contribution.

IV. TREE AND LOOP CONTRIBUTIONS

When combining the tree and one-loop contributions to obtain the muon helicity flip and non-flip amplitudes, we can ignore the tree-one-loop interference terms because of the suppression discussed in the previous section. Consequently, the combined contributions from the tree and one-loop amplitudes to the squared spin-summed amplitude, in the Eq. (1),

$$\sum_{\text{spin}} |\mathcal{M}|^2 = \sum_{\text{spin}} |\mathcal{M}_{\lambda\lambda\lambda\gamma}^{\text{tree}} + \mathcal{M}_{\lambda\lambda\lambda\gamma}^{\text{loop}}|^2, \quad (28)$$

can be simplified to

$$\sum_{\text{spin}} |\mathcal{M}|^2 = \sum_{\text{spin}} |\mathcal{M}_{\lambda\lambda\lambda\gamma}^{\text{tree}}|^2 + \sum_{\text{spin}} |\mathcal{M}_{\lambda\lambda\lambda\gamma}^{\text{loop}}|^2, \quad (29)$$

where

$$\begin{aligned} \sum_{\text{spin}} |\mathcal{M}_{\lambda\lambda\lambda\gamma}^{\text{tree}}|^2 &= |\mathcal{M}_{--++}^{\text{tree}}|^2 + |\mathcal{M}_{++--}^{\text{tree}}|^2 \\ &\quad + |\mathcal{M}_{++++}^{\text{tree}}|^2 + |\mathcal{M}_{----}^{\text{tree}}|^2, \end{aligned} \quad (30)$$

$$\begin{aligned} \sum_{\text{spin}} |\mathcal{M}_{\lambda\lambda\lambda\gamma}^{\text{loop}}|^2 &= |\mathcal{M}_{-++}^{\text{loop}}|^2 + |\mathcal{M}_{+--}^{\text{loop}}|^2 \\ &\quad + |\mathcal{M}_{-+-}^{\text{loop}}|^2 + |\mathcal{M}_{-+-}^{\text{loop}}|^2. \end{aligned} \quad (31)$$

Using Eq. (29), we may write the following relations for the $\mu\bar{\mu}$ -invariant mass distributions and the decay widths

$$\frac{d\Gamma}{dm_{\mu\bar{\mu}}} = \frac{d\Gamma^{\text{tree}}}{dm_{\mu\bar{\mu}}} + \frac{d\Gamma^{\text{loop}}}{dm_{\mu\bar{\mu}}}, \quad (32)$$

$$\Gamma = \Gamma^{\text{tree}} + \Gamma^{\text{loop}}. \quad (33)$$

In Fig. 3, we show $\mu\bar{\mu}$ -invariant mass distributions for the decay $H \rightarrow \mu\bar{\mu}\gamma$. This figure illustrates the tree and one-loop contributions to the invariant mass distributions $d\Gamma^{\text{tree}}/dm_{\mu\bar{\mu}}$ and $d\Gamma^{\text{loop}}/dm_{\mu\bar{\mu}}$, respectively. The complete distribution, $d\Gamma/dm_{\mu\bar{\mu}}$, according to the Eq. (32), is simply the sum of these two contributions. Fig. 4, shows the tree and one-loop contributions, Γ^{tree} and Γ^{loop} , as well as their sum, Γ . For comparison, the widths of the tree level decay $H \rightarrow \mu\bar{\mu}$ the decay $H \rightarrow \gamma\gamma$ are included.

V. SUMMARY

We have shown that, for $m_H \lesssim 130$ GeV, the contributions from the tree and loop levels to the decay $H \rightarrow \mu\bar{\mu}\gamma$ are comparable. It is, therefore, necessary to include both contributions to the $\mu\bar{\mu}$ -invariant mass distribution and the decay width. In this mass region, as elsewhere, the interference terms between the tree and loop amplitudes are small compared to the squared amplitude of either one, and the total decay width is simply the sum of the decay widths from the tree and one-loop contributions, $\Gamma = \Gamma^{\text{tree}} + \Gamma^{\text{loop}}$. When $m_H \gtrsim 130$ GeV, the one-loop contribution dominates, and, for m_H larger than 140 GeV, it exceeds the tree level contribution to $H \rightarrow \mu\bar{\mu}$.

Finally, the presence of the top quark loop in some of the diagrams in the Fig. 1(a) offers an opportunity to use the decay $H \rightarrow \mu\bar{\mu}\gamma$ as a possible probe of the Higgs boson coupling to the top quark. In Fig. 5, we show the decay width for $H \rightarrow \mu\bar{\mu}\gamma$ that arises from the complete set of diagrams in Fig. 1 after modifying the $Ht\bar{t}$ coupling. Modification has been achieved by multiplying the Standard Model $Ht\bar{t}$ coupling by a factor λ [7].

As it can be seen from the Fig. 5, the higher the Higgs boson mass is, the more difficult it is to distinguish the Standard Model coupling for $Ht\bar{t}$, $\lambda = 1$, from the case of no coupling between the Higgs boson and the top quark, $\lambda = 0$. Generally, a measured value of the decay width Γ corresponds to two different values of λ . Therefore, the determination of λ will not be unique. However, it can be used as an indication of a deviation from the Standard Model.

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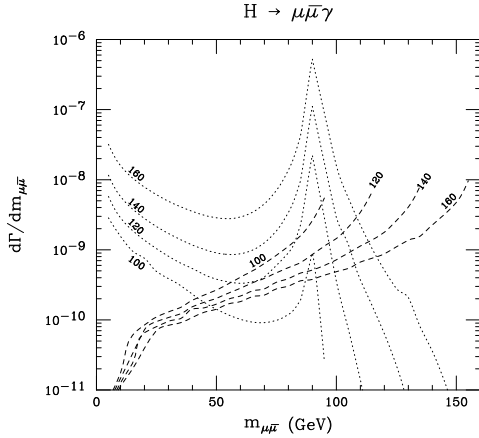
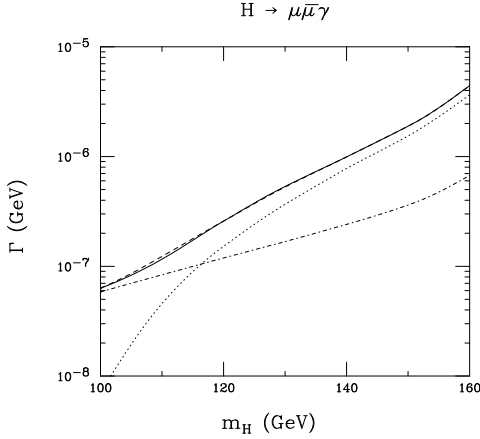
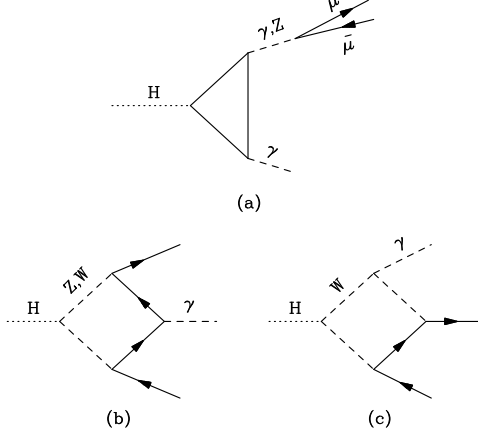


FIG. 3. The $\mu\bar{\mu}$ -invariant mass distribution of the decay $H \rightarrow \mu\bar{\mu}\gamma$, for Higgs boson masses of 100 GeV, 120 GeV, 140 GeV, and 160 GeV is shown. The dotted lines are for the loop contribution and dashed lines are for the tree level contribution. The combined contribution is simply the sum of the tree and the loop contributions.

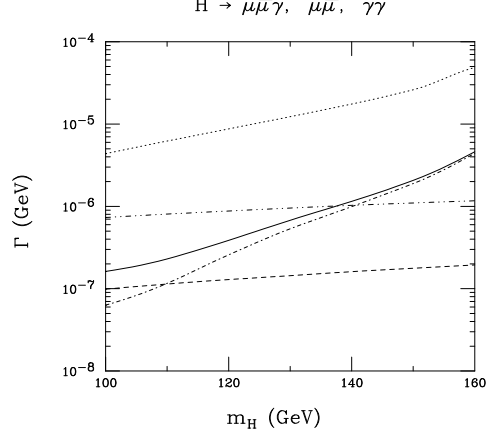


FIG. 4. The decay width for several decay modes of the Higgs boson is shown. The dashed line is the width for the decay $H \rightarrow \mu\bar{\mu}\gamma$ at tree level, the dot-dashed line is the width at the one-loop level, the solid line is the total width (the sum of the dashed and dot-dashed lines), the dot-dot-dashed line is $\Gamma(H \rightarrow \mu\bar{\mu})$ at tree level, and the dotted line is $\Gamma(H \rightarrow \gamma\gamma)$.

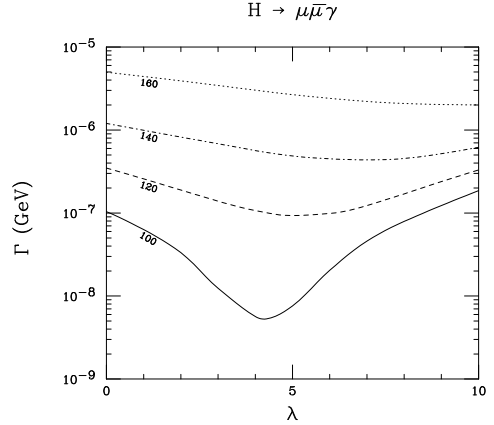


FIG. 5. The decay width $\Gamma(H \rightarrow \mu\bar{\mu}\gamma)$, at the loop level, as a function of the $Ht\bar{t}$ coupling (in multiples λ of the Standard Model coupling), for several values of Higgs boson mass is shown. The solid line is $m_H = 100$ GeV, the dashed line is $m_H = 120$ GeV, the dot-dashed line is $m_H = 140$ GeV, and the dotted line is $m_H = 160$ GeV.